Algebra II
Dr. Paul L. Bailey

Homework 1123
Tuesday, November 23, 2021

Name:

Due Wednesday, November 24, 2021. Write all complex numbers and polynomials in standard form. Do not copy. Do not write anything you do not understand.

Definition 1. Let A and B be sets. The *union* if A and B, denoted $A \cup B$, is the set consisting of all the elements that are in either A or in B. That is,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

Example 1. The set of all real numbers except 0 is the set of all negative numbers, union with all the positive numbers. The set of negative numbers is $(-\infty, 0)$. The set of positive numbers is $(0, \infty)$. Therefore, the set of all real numbers except 0 can be written in these three ways:

$$\{x \in \mathbb{R} \mid x \neq 0\} = \{x \in \mathbb{R} \mid x < 0 \text{ or } x > 0\} = (-\infty, 0) \cup (0, \infty).$$

Problem 1. Write the following subsets of $\mathbb{R}$ using correct set notation.

(a) The set of positive integers less than or equal to 7.

(b) The set of real numbers greater than 3.14.

(c) The set of real numbers except 3.

(d) The set of real numbers whose square is less than 25.

(e) The set of real numbers whose square is greater than 9.

(f) The set of real numbers x such that $x^3 - x > 0$.

Problem 2. Let $f(x) = x^3 - 2x^2 - 9x + 18$.

(a) Find the zeros of f. You may use "factor by grouping".

(b) Draw a sign chart for f.

(c) Solve the inequality $x^3 + 18 \geq 2x^2 + 9x$.

Problem 3. Let $f(x) = x^3 - 9x^2 + 33x - 65$.

(a) Use synthetic division to show that $f(5) = 0$.

(b) Let $q(x)$ be the quotient when $f(x)$ is divided by $x - 5$, so that $f(x) = (x - 5)q(x)$. Write $q(x)$ in standard form.

(c) Use the quadratic formula to solve $q(x) = 0$. Simplify.

(d) Solve $f(x) = 0$. Write the solution set.